Title	Secondary 1 ~ 4 Elementary Mathematics Material Compilation
Editor	Lee Jian Lian
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Title	Basic Algebra Notes
Author	Lee Jian Lian
Quality Control	Liu Hui Ling and Lim Wang Sheng

Algebra is basically

- Representing numbers with letters and symbols
- Performing mathematical operations using letters and symbols, together with using numbers.

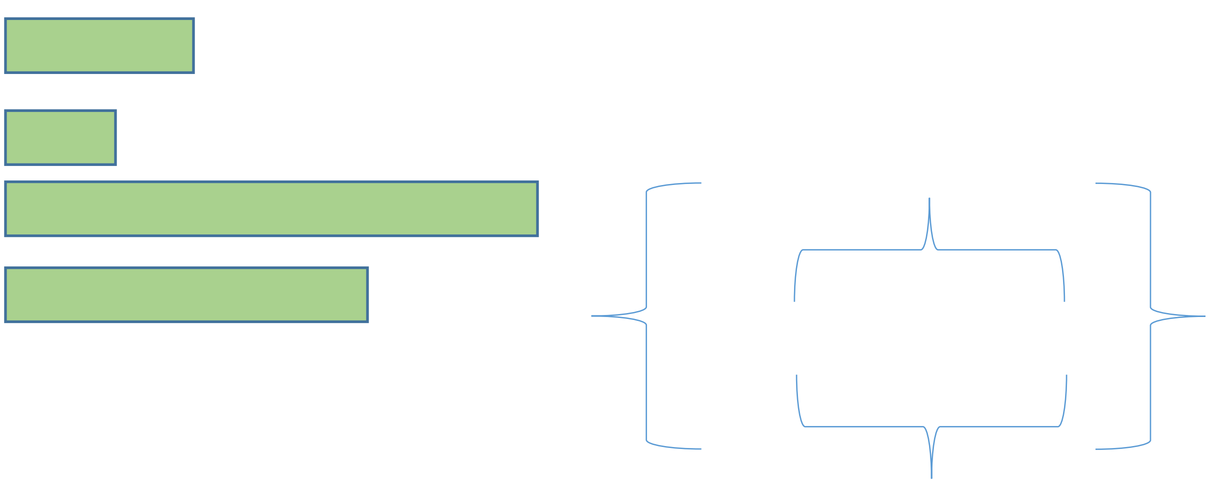
If algebra never existed

- Textbooks will be ridiculously heavy
- Statistical analysis will be impossible to do, as the person writing the data will have virtually endless amount of explanation he/she got to make.
- Certain fields of mathematics, dependent on algebra, cannot exists as well.

Tips to students who are new to algebra:

- Name your variables systematically, in a way you won't get confused
- Don't use letters and symbols that can easily be confused for something else, (I never use Q, O, Z, J, I, S and E, unless the question specify I must use them.)
  [If you go to higher level mathematics, you will understand why E and I are rarely used in mathematical variable representation, they are reserved for specific use.]
- If you ever had to use Z, please strike through the letter in the following manner:
  ~~Z~~
  [To avoid confusion with the number "2"]
- Use visualization to connect model drawings with algebra representation.

For convenience in typing later: I have decided to create a few bars here so they can be copied with a few clicks and on demand. (I don't need to draw again and again, which is tedious even on a computer.) (Teachers with issue with using the computer or word processor can just copy my template and use them.)



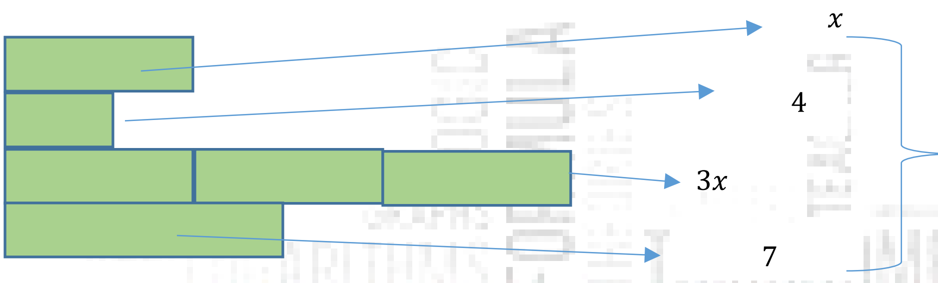
Basic Operands in Algebra:	
Expression	English Meaning
$x + 5$	Add 5 to x
$x - y$	Subtract y from x
$6n = n + n + n + n + n + n$	Multiply n by 6
$\frac{m}{n}$	Divide m by n

Exponents and Roots	
Expression	English Meaning
$n^7 = n \times n \times n \times n \times n \times n \times n$	n to be multiplied by itself 7 times.
$n^{-5} = \frac{1}{n^5} = \frac{1}{n \times n \times n \times n \times n}$	1 divided by total value of n multiplied 5 times in a row.

Example 1:
Solve the following Expression
$x + 7 + 3x + 4$

$= x + 3x + 7 + 4$
$= 4x + 11$

Let's convert it to models and see how they corelate:



Sum up all the values represented by the models and you should get the same expression.

How algebra is translated into models in this case:

1. There are four boxes of the variable x which is equal to $4x$.
2. There are one box of 7 and one box of 4, hence the total value is $4x + 11$.

The above also, in a way, demonstrate the concept of “like” and “unlike” terms.
In any algebraic addition and subtraction, only “like terms” can be added or subtracted in this way.

$2x + 3x = 5x$
$5x - 4x = x$

Arithmetic demonstration of concept:
$2x + 3x = 5x$

Since $2x = 2 \times x$,
We can pretend the value of x is 5.

In this case:
$2x = 2 \times 5$ and $3x = 3 \times 5$
$10 + 15 = 25$

The other way is true as well and valid:
$2x + 3x = 5x = 5 \times 5 = 25$

More Complicated Subtraction and Addition and Demonstration by Purely Algebra Approach and How to Check Your Answers.

Example 2:

$3x + 5 - 3x + 4$

$= 3x - 3x + 5 + 4$ (Group "Like Terms" together)

$= 0 + 9 = 9$ (Final Answer)

To verify your answer is correct, what you need to do.

Pretend $x = 2$

$(3 \times 2) + 5 - (3 \times 2) + 4$

$= 6 + 5 - 6 + 4$

$11 - 6 + 4 = 5 + 4 = 9$

Example 3:

$15xy - 5 + 2 - 13xy$

$= 15xy - 13xy - 5 + 2$ (Group "Like Terms" Together)

$= 2xy - 5 + 2$

$= 2xy - 3$ (Final Answer)

To verify the correctness of your answer, you need to

Pretend $x = 2$ and $y = 3$ in your original question.

$(15 \times 2 \times 3) - 5 + 2 - (13 \times 2 \times 3)$

$(90) - 5 + 2 - (78)$

$= 90 - 5 + 2 - 78 = 9$

Then pretend $x = 2$ and $y = 3$ in your answer.

$(2 \times 2 \times 3) - 3$

$= 12 - 3$

$= 9$

Some teachers also call this type of answer checking as "Checking Answers by Value Substitution" as you are indeed "Substituting the Value" of the letters by another value.

Problem Solving (Addition and Subtraction of Algebra):

To solve a problem sum using algebra, while it varies slightly in different questions, we observed the following steps are necessary.

1. Find out what are the unknowns, needed to solve the question.
2. Define the known and unknown variable properly using algebra letters and numbers.
3. Construct the equation or expression needed to solve the question.
4. Solve the equation and find the value of unknown variable.

The easiest example I've seen on books that can demonstrate this:
Example 4:
The sum of three consecutive numbers is 84. What is the smallest of the three numbers.

1	Find out the unknown.	The smallest number out of the three consecutive numbers.
2	Define the unknown and known properly	In this case, the unknown value (smallest number) is defined as x. Since they said the numbers are in consecutive order, we can define the subsequent two numbers as $x + 1$ and $x + 2$
3	Construct an Equation	Sum means, add the three numbers together, so we have: $x + (x + 1) + (x + 2) = 84$
4	Solve the Equation and find out what is the unknown value, in this case, it is x.	$3x + 1 + 2 = 84$ $3x + 3 = 84$ $3x = 84 - 3$ $3x = 81$ $x = 27$ [Final Answer] Thus, the smallest number is 27.

Performing Algebra Addition and Subtraction (Using Brackets) Demonstration and Concept:

The below two example seems extremely straightforward:
$a + b = a + b$
$a - b = a - b$

It is the below few example students often get confused about and are careless enough to lose a total of 7 marks in exams.

$a + (-b) = a - b$
$a - (-b) = a + b$

$(-a) + b = b - a$
$(-a) + (-b) = (-b) - a$

Performing Algebra Multiplication and Division (Using Brackets) Demonstration and Concept:

The below two examples are straightforward.
$a(b) = ab$
$a \div b = \frac{a}{b}$

Once again students can get really confused when negative signs are involved.
$a(-b) = -ab$
$-a(b) = -ab$
$(-a)(-b) = ab$

$a \div (-b) = -\frac{a}{b}$

$(-a) \div b = -\frac{a}{b}$

$(-a) \div (-b) = \frac{a}{b}$

Example 5:
Simplify $-2(3x - 4 + 6x)$

We must take note that the number to be multiplied in is a negative number, and that there is a bracket that contains the expression $3x - 4$.

$-2(3x) - 2(-4) - 2(6x)$	Remove the Brackets by Multiplying the Value outside the Bracket with the Value inside the bracket.
$= -6x + 8 - 12x$ $= -6x - 12x + 8$	Group Like Terms Together
$-18x + 8$	Final Answer

Example 6:
Simplify $\frac{1}{3x-1} + \frac{3x}{5}$
This time, we are doing mathematics on something called "Algebra Fractions". You may be thinking, how are you supposed to do such a question. I would like to tell you, exactly in the same way as how you would do your fractions in primary school.

Recap from Primary School Mathematics:
Evaluate $\frac{2}{5} + \frac{1}{4} =$

$$\frac{2}{5} + \frac{1}{4}$$

After cross multiplying, you should get something like below:

$$\frac{4(2)}{20} + \frac{5(1)}{20} = \frac{8+5}{20} = \frac{13}{20}$$

In case of $\frac{1}{3x-1} + \frac{3x}{5} =$

$$\frac{1}{3x-1} + \frac{3x}{5} =$$

$$\frac{5(1)}{5(3x-1)} + \frac{3x(3x-x)}{(3x-1)(5)} =$$

$$\frac{5}{15x-5}+\frac{9x^2-3x^2}{15x-5}=$$

$$\frac{5+6x^2}{15x-5}$$

Example 7
(Dealing with negative signs in algebra fractions)

Simplify $\frac{2x}{9}-\frac{2x-5}{7}$

In this case, do note the sections highlighted in yellow.

After cross multiplying, you should get something like the below expression.

$$\frac{7(2x)}{63}-\frac{9(2x-5)}{63}$$

$$=\frac{14x-18x+45}{63}$$

$$=\frac{-4x+15}{63}$$

Best Practices:

- Fraction should be combined into single denominator right after cross multiplication to prevent confusion.
- Bracket should be used all the time when cross multiplying terms to maintain consistency thus, preventing confusion, once again.

Example 8:
(Dealing with a type of algebraic expression where you have to deal with both whole number coefficients and algebra fractions in the same expression)

First, let me explain what I mean by dealing with both whole number coefficients and fraction within one a single row.

It refers to an algebraic expression that resembles the following expressions.

$$\frac{a}{b} + nc$$

Where n is the coefficient of c and n is a whole number

To make it even clearer, let me demonstrate a proper question below.

Simplify $\frac{5x+6}{5} - 4x$

In this case, we just need to multiply the $-4x$ by 5.
The above expression is then transformed into $\frac{5x+6}{5} - \frac{5(4x)}{5(1)}$

Rationale:
For the sake of putting both the expression within the same denominator, we need to create a denominator for $4x$.

Since $4x = \frac{4x}{1}$, we can convert the value to $\frac{4x}{1}$.

And because the left side of the expression shows $\frac{5x+6}{5}$, we need to cross multiply the right ride of the expression by 5 as well, giving us a value of $\frac{5(4x)}{5} = \frac{20x}{5}$.

This means, the expression is now being written with a common denominator and can be rewritten as:
$$\frac{5x+6-20x}{5}$$

Which can be simplified into:
$$\frac{-15x+6}{5}$$

Title	Basic Algebra – Changing Subject of Formula [Final Version]
Author	Lim Wang Sheng, School of Information Technology, Nanyang Polytechnic [Part of NYP Mentoring Club]
Date	13/3/2018

Realizing in my class, a lot of students have issues with algebra and I anticipate similar problems to occur with its slightly trickier counterpart, formula manipulation. I decided to write some materials for my own students to use. Due to involvement in public projects, I decided to publish this file through various people, hoping it will help other students as well.

Definitions

Changing Subject of Formula – Making an algebraic symbol in a formula (anything, from letters, symbols) appear as a standalone variable on the left-hand side of the formula. [Will be described in greater detail and will explain how my definition works]

Notation used in this document	
Notation	**Meaning**
RHS	Right-Hand Side of a formula
LHS	Left-Hand Side of a formula

Important things to take note (Or be cautious about)

1. Squares and Square Roots (Negative and Positive Answers are possible in some cases, will be demonstrated in the subsequent pages)
2. (Rarely) If they specify the subject of formula should be a variable that only has a positive value (Example include, distance and speed), be aware that the negative value should be rejected in this case as distances and speed cannot have any negative signs.

Explaining Point 1 in Greater Detail Using Arithmetic:

Supposed we have the number "4" and we want to square root the number.

$\sqrt{4} = 2 \; or - 2$

Both 2 and (-2) satisfy the requirement of being a square root value of 4,

$2^2 = 4$ AND $(-2)^2 = 4$

Example 1:
Make c the subject of the formula.
$E = mc^2$

When they ask you to make c the subject of the formula, they are asking you to rearrange the formula in a way that c must appear as a single algebraic letter on the LHS of the formula.

Steps Taken	Explanation
$\frac{E}{m} = c^2$	Divide both sides by m to isolate c^2
$c^2 = \frac{E}{m}$	Change c^2 to the LHS of the formula
$c = \pm\sqrt{\frac{E}{m}}$	Square rooting c^2 on the LHS gives two possible values, positive and negative, on the right-hand side of the formula.

Example 2:
Make $\boldsymbol{m}$ the subject of the formula
$E = \frac{1}{2}mv^2$

Steps	Explanation
$2(E) = mv^2$	Get rid of fraction by multiplying both sides by 2.
$mv^2 = 2E$	Change sides of the formula
$m = \frac{2E}{v^2}$	Divide both sides by v^2 to isolate and get m

Example 3

Make the b the subject of the formula (Not taken from physics, self-created)

$\frac{3}{f}+\frac{1}{bc}=\frac{1}{g}$

Steps	Explanation
$\frac{1}{bc}=\frac{1}{g}-\frac{3}{f}$	Isolate $\frac{1}{bc}$ from formula
$\frac{1}{bc}=\frac{1(f)-3(g)}{fg}$	Create a common denominator for RHS
$\frac{1}{bc}=\frac{f-3g}{fg}$	
$bc=\frac{fg}{f-3g}$	When a fraction gets inversed on the LHS, the same applies to the RHS. Can be explained in the following manner: $\left(\frac{1}{bc}\right)^{-1}=\left(\frac{f-3g}{fg}\right)^{-1}$ $bc=\frac{fg}{f-3g}$
$b=\frac{(fg)}{c(f-3g)}$	Divide both sides by c to isolate b on the LHS

Example 4:
Make v the subject of the formula.

$$t = \left(\frac{s}{\sqrt{1 - \frac{v^2}{c^2}}}\right)$$

Steps taken	Explanation
$t^2 = \frac{s^2}{1 - \left(\frac{v^2}{c^2}\right)}$	Removing any square root signs in the formula by squaring both sides of the formula
$t^2\left(1 - \frac{v^2}{c^2}\right) = s^2$	Cross Multiply
$t^2 - \frac{tv^2}{c^2} = s^2$	Removal of brackets
$-\frac{tv^2}{c^2} = s^2 - t^2$	Minus "t^2" from both sides
$-tv^2 = c^2(s^2 - t^2)$	Cross Multiply
$v^2 = \frac{c^2(s^2 - t^2)}{-t}$	Divide both sides by $-t$ to isolate and get v^2
$v = \pm\sqrt{\frac{c^2(s^2 - t^2)}{-t}}$	v is the subject of the formula, square rooting the LHS gives two possible values on the RHS, positive and negative.

Example 5:

Given the following formula,

$t = \frac{n}{\sqrt{1-\frac{2GM}{rc^2}}}$

Make c the subject of the formula.

Steps taken	Explanation
$t^2 = \frac{n^2}{1-\frac{2Gm}{rc^2}}$	Remove Square Root
$t^2\left(1-\frac{2Gm}{rc^2}\right) = n^2$	Cross Multiply
$t^2 - \frac{2t^2Gm}{rc^2} = n^2$	Remove Brackets
$-\frac{2t^2Gm}{rc^2} = n^2 - t^2$	Minus t^2 from both sides
$-2t^2Gm = rc^2(n^2 - t^2)$	Cross Multiply
$rc^2 = \frac{-2t^2Gm}{(n^2 - t^2)}$	Divide both sides by $(n^2 - t^2)$
$c^2 = \frac{-2t^2Gm}{r(n^2 - t^2)}$	Divide both sides by r to isolate and get c^2
$c = \pm\sqrt{\frac{-2t^2Gm}{r(n^2 - t^2)}}$	Square root the value on both sides, once again, square rooting the LHS gives two possible value on the RHS.

Title	Basic Concept of Inequality in Mathematics (Secondary 1E/1NA/2NT)
Author	Lim Wang Sheng, School of Information Technology, Nanyang Polytechnic [NYP Mentoring Club]
Notes	This is designed for use by students from every stream (NA/E/NT)

Refer to This if You are confused!

Symbols Used	Meaning
$>$	Greater Than
$<$	Less Than

Applications of inequalities ranges from

- Science (Physics, Engineering and Chemistry, to explain that certain science or engineering related situation will only occur at a given range of values)
- Computing and Game Development (To code computer game and program logic)

Follow the following rules when solving inequalities	
If you add, subtract, multiply or divide on one side of the inequality by any values, do the same on the other side. If you add, subtract any types of values (positive or negative) on both sides, the signs are unchanged. If you multiply or divide positive values on both sides, the inequality signs are unchanged.	Example 1: $m + 5 > 10$ $m + 5 - 5 > 10 - 5$ Example 2: $m - 5 > 10$ $m - 5 + 5 > 10 + 5$ $m > 15$ Example 3: $6m > 15$ $\frac{6m}{6} > \frac{15}{6}$ $m > \frac{15}{6}$ $m > \frac{5}{2}$
If you multiply or divide negative values on both sides, the inequality signs <u>must</u> change.	Example 4: $-m > 5$ $(-1)(-m) < (-1)(5)$ $m < 5$

	Example 5: $-10m > 5$ $-\frac{10m}{-10} < \frac{5}{-10}$ $m < -0.5$

Further Proof of Concept Given Below Using Numbers	
Adding of Positive Numbers	$5 > 2$ $5 + 4 > 2 + 4$ $9 > 6$ 9 is still greater than 6
Subtracting of Positive Values	$10 > 2$ $10 - 2 > 2 - 2$ $8 > 0$ 8 is still greater than 0
Multiplication of Positive Values	$16 > 1$ $16 \times 2 > 1 \times 2$ $32 > 2$ 32 is still greater than 2
Division of Positive Values	$12 > 9$ $\frac{12}{3} > \frac{9}{3}$ $4 > 3$ 4 is still greater than 3
Adding of Negative Numbers	$-2 < 5$ $-2 + (-5) < 5 + (-5)$ $(-2) - 5 < 5 - 5$ $-7 < 0$ -7 is still less than 0
Subtracting of Negative Values	$-18 < 6$ $-18 - (-4) < 6 - (-4)$ $-18 + 4 < 6 + 4$ $-14 < 2$ -14 is still less than 2

*Multiplication of Negative Values	$-15 > -22$ $-1(-15) < -1(-22)$ $15 < 22$ After multiplying -1 to the inequality, 15 is less than 22
*Division of Negative Values	$-30 < 1$ $\frac{-30}{-5} > \frac{1}{-5}$ $\frac{30}{5} > 0.2$ $6 > 0.2$ After dividing -5 to the inequality, 6 is greater than 0.2

Introducing "Greater Than or Equals to" and "Less Than or Equals to"

Refer to this if you are confused

Symbols Used	Meaning
$\geq$	Greater Than or Equals To
$\leq$	Less Than or Equals to

The way the "Greater than or Equals to" and "Less Than or Equals to" work is similar to examples I wrote in the last few pages.

Just a few things I hope students take note of

(Common sense, but well, some students are that careless)

- If the question didn't mention symbols "$\leq$" OR "$\geq$", you don't go and write those symbols down, using incorrect symbols results in loss of marks.

If question mentions symbols "$\leq$" OR "$\geq$", similar rules applies

- If you add or subtract any numbers from the inequality, the symbols are unchanged.
- If you multiply or divide positive numbers from the inequality, the symbols are unchanged
- If you multiply or divide negative numbers from the inequality,
  $\geq$ changes to $\leq$
  $\leq$ changes to $\geq$

Example 1	Solve the following inequality $x + 6 \geq 12$ $x + 6 - 6 \geq 12 - 6$ $x \geq 6$
Example 2	Solve the following inequality $y - 6 \leq 42$ $y - 6 + 6 \leq 42 + 6$ $y \leq 48$
Example 3	Solve the following inequality $\frac{m}{2} \geq 5$ $2\left(\frac{m}{2}\right) \geq 2(5)$ $m \geq 10$
Example 4	Solve the following inequality $7m \geq 14$ $\frac{7m}{7} \geq \frac{14}{7}$ $m \geq 2$

Example 5***	Solve the following inequality $-\frac{m}{5} \leq 15$ $-5\left(-\frac{m}{5}\right) \geq -5(15)$ $m \geq -75$ ***Note how the sign changes.
Example 6***	Solve the following inequality $-7m \geq 28$ $\frac{-7m}{-7} \leq \frac{28}{-7}$ $m \leq -4$ ***Note how the sign changes.

Title	Mathematics (Lowest Common Multiple and Highest Common Factor)
Author	Lim Wang Sheng, School of Information Technology, Nanyang Polytechnic [Mentoring Club, Nanyang Polytechnic] (Editor) Lee Jian Lian
Date	24/2/2018

Understanding some schools chose to teach the law of indices in Secondary 3 rather than Secondary 1, I will like to explain various ways on how the laws of indices works before proceeding to introduce the concepts as it will be frequently used in the documents.

Notation	English	Explanation
a^b Example: $3^2 = 3 \times 3$ $2^3 = 2 \times 2 \times 2$	a to the power of b	It means to multiply a by itself b number of times. a is the base while b is the power or the exponent.

Law of Indices Relevant to this topic

$(a^b)(a^c) = a^{b+c}$

$\frac{(a^b)}{(a^c)} = a^{b-c}$

$(a^b)^c = a^{bc}$

In General

$\sqrt[c]{a^b} = a^{\frac{b}{c}}$

Specifically

$\sqrt{a} = a^{\frac{1}{2}}$

$\sqrt[3]{a} = a^{\frac{1}{3}}$

Definitions:

A prime number is a number greater than 1, that can only be divisible by itself or 1.

Recalling how you find Lowest Common Multiple and Highest Common Factor in Primary School.

Example 1:
Find the Lowest Common Multiple and Highest Common Factor of 18 and 15.

Highest Common Factor
(Listing down all factors and drawing conclusion of the highest factor shared between both values.)

Numbers	Arithmetic Form	Listed
18	1×18 2×9 6×3	1,2,3,6,9,18
15	1×15 3×5	1,3,5,15

The highest common factor is deduced to be 3.

Lowest Common Multiple
(Listing down every multiple possible for both values and to draw conclusion of the lowest multiple shared between both values)

Numbers	Listed	
18	18,36,54,72,90,108	
15	15,30,45,60,75,90,105	

The lowest common multiple is therefore 90.

The problem with this method, it works only for small numbers, as the numbers get larger and larger, listing the factors and multiples becomes increasingly difficult and tiring and thus we need a more efficient method to find lowest common multiples and highest common factors.

Example 2:
Find the highest common factor of 168 and 560.

Step 1: Divide both numbers repeatedly by prime numbers until you get 1 and thus unable to divide further.

Divisor	Dividend	Quotient
2	560	280
2	280	140
2	140	70
2	70	35
5	35	7
7	7	1

Divisor	Dividend	Quotient
2	168	84
2	84	42
2	42	21
7	21	3
3	3	1

Step 2: Express Numbers in Index Notation.
$560 = 2^4 \times 5 \times 7$
$168 = 2^3 \times 3 \times 7$

Step 3: Highlight values with common bases.
$560 = 2^4 \times 5 \times 7$
$168 = 2^3 \times 3 \times 7$

Step 4: Take value which satisfy two conditions: Highlighted (Common Base), Lowest Power of the two rows.

What we get is	2^3 and 7

Step 5:
Multiply the values to get Highest Common Factor

HCF = $2^3 \times 7 = 56$

Example 3:
Find the lowest common multiple of 45 and 378.

Step 1: Divide the values repeatedly by prime numbers until you get 1, thus unable to divide any further.

Divisor	Dividends	Quotients
3	45	15
3	15	5
5	5	1

Divisor	Dividends	Quotients
2	378	189
3	189	63
3	63	21
7	21	3
3	3	1

Step 2: Express Numbers in Index Notation.
$45 = 3^2 \times 5$
$378 = 2 \times 3^3 \times 7$

Step 3: Highlight values that satisfy the following condition.
If base common in both rows, pick the one with highest power.
If the base value is only found on either row, highlight the value as well.

What we get is	$2, 5, 3^3, 7$

Step 4: Multiply the highlighted values together to get Lowest Common Multiple.

LCM = $2 \times 3^3 \times 5 \times 7$ =1890

Example 4:

(a) Express 2700 as a product of its prime factors.
(b) Given that 2700h is a perfect square, write down the smallest possible value of h
(c) Given that 2700k is a perfect cube, write down the smallest possible value of k.

4(a)

Divisor	Dividends	Quotient
2	2700	1350
2	1350	675
5	675	135
5	135	27
3	27	9
3	9	3
3	3	1

Product of Prime Factors (Index Notation): $2^2 \times 5^2 \times 3^3 = 2700$

4(b)
To solve this question, I would like to introduce you to the vocabulary needed to solve the question and the rules of solving similar questions.

For a number to be even called a perfect square, it needs to:

- Produce an integer when square rooted.

For 2700h to be a perfect square, it needs to satisfy the following requirements:
Multiplying h to 2700 should be able to ensure all the power of the prime factors of 2700h are divisible by 2.

Which bring us to the law of indices

$a^b \times a^c = a^{b+c}$

Apparently, since the only base value where the power isn't divisible by 2 is 3^3
And that $(3^3)(3) = 3^4$

$2700h = 2^2 \times 5^2 \times 3^3 \times 3 = 2700(3)$
$h = 3$ (Final Answer)

4(c)

For a number to be called a perfect cube, it needs to

- Produce an integer when being cube rooted.

For 2700k to be a perfect cube in this case, multiplying k to 2700 results in:

- Powers of prime factors of 2700k should all be divisible by 3.

As you see from the following values, there are two values for which the powers are not divisible by 3.

$2^2 \times 5^2 \times 3^3 = 2700$

Since $2^2 \times 2 = 2^3$
And $5^2 \times 5 = 5^3$

$2700k = 2(2^2) \times 5(5^2) \times 3^3 = 2^3 \times 5^3 \times 3^3$

$k = 2 \times 5 = 10$ (Final Answer)

(Questions all obtained from CASCO Mathematics 4B Assessment Book)

Title	Mathematics Coordinate Geometry (Lower Secondary)
Editor	Lee Jian Lian
Date	1/4/2018

Formula List Required

General Form (or General Equation) of a Straight Line:

$y = mx + c$

Where m is the gradient and c is the y-intercept (Or the value of y when $x = 0$)

[If line is vertical, the gradient is undefined.]

Given Coordinates (x_1, y_1) AND (x_2, y_2) are coordinates on a straight line.

Gradient = $\frac{y_2 - y_1}{x_2 - x_1}$

Length of Line (In Units) = $\sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$

Basic Steps Briefing

To find the equation of a line, you need to do the following steps:

1. Write down the coordinates of points where the line will pass through
2. Calculate gradient by using the above mentioned gradient formula
3. Substitute coordinate into the general equation form of a straight line to find out the y-intercept.
4. Collect Gradient Information and Y-Intercept to deduce equation
5. (If applicable) Make use of deduced equation to solve any remaining questions

Notes:

If gradient is undefined but the question insist that you should write an equation for the line, you can write the equation of the line in terms of x. (e.g. $x = 5$)

[Questions all Taken from CASCO 4B Assessment Book]

Example 1

The coordinates of A and B are $(3, -5)$ and $(-1, -9)$ respectively.

Find

(a) the Length of AB.

(b) the equation of Line AB.

(c) the value of p if the point $C(2p, 6)$ lies on the line AB

(d) the equation of the line l which has the same gradient as the line $3x + y = 1$ and passing through point C.

<table>
<tr><th>Parts</th><th>Formula and Reasoning</th><th>Steps Taken</th></tr>
<tr><td>(a)</td><td>Length of Line (In Units) = $\sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$</td><td>
<table>
<tr><td>y_2</td><td>y_1</td><td>x_2</td><td>x_1</td></tr>
<tr><td>-9</td><td>-5</td><td>-1</td><td>3</td></tr>
</table>
Value of $y_2 - y_1 = (-9) - (-5) = -4$

Value of $x_2 - x_1 = (-1) - 3 = -4$

$\sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$ = $\sqrt{(-4)^2 + (-4)^2} = \sqrt{32} = 5.6569 = 5.66\ units\ (3sf)$
</td></tr>
<tr><td>(b)</td><td>Gradient = $\frac{(y_2 - y_1)}{(x_2 - x_1)}$</td><td>
<table>
<tr><td>y_2</td><td>y_1</td><td>x_2</td><td>x_1</td></tr>
<tr><td>-9</td><td>-5</td><td>-1</td><td>3</td></tr>
</table>
Value of $y_2 - y_1 = (-9) - (-5) = -4$
Value of $x_2 - x_1 = (-1) - 3 = -4$

Therefore gradient, $m = 1$

$y = mx + c$
</td></tr>
</table>

	Substitute associated coordinates into the graph equation to find y-intercept.	You can see how it resembles and correspond to the coordinates in the form of: $y_2 = m(x_2) + c$ $(-9) = 1(-1) + c$ $(-9) - 1 = c$ $c = -8$ Thus, equation of the Line AB is: $y = x - 8$
(c)	Substitute associated coordinate into the equation to find the value of x, when $x = 2p$ and $y = 6$	The equation is once again, in the form: $y = x - 8$ $6 = x - 8$ $6 + 8 = x$ $2p = x = 14$ $p = 7$
(d)	Line L has the same gradient as $3x + y = 1$ At point $C(14,6)$	Rearrange in the form of $y = mx + c$ $y = -3x + 1$ Gradient of Line $L = -3$ $y = mx + c$ $6 = -3(14) + c$ $c = 48$ Thus, Equation of Line L is $y = -3x + 48$

Title	Mathematics (Direct & Inverse Proportion)
Date	22/3/2018
Author	Lee Jian Lian

This material is designed to teach students basics of inverse and direct proportion.

The following table describes the notation I used when teaching my friend's siblings about this topic, you can consider using my notation or consult your teachers about what notation to use.

Notation	English Name	Meaning
$(Subscript)_{Given}$	"Subscript Given Value"	Values derived from given information in the question.
$(Subscript)_{New}$	"Subscript New Value"	Values used to solve later parts of the question.

Please also state which part of the question you are trying to solve in the question paper before writing down how you solve the question. I will also demonstrate how I use those notation, you will realize how clear my working is. Students with problems with producing clear workings can consider a similar way of writing down workings like me.

Direct Proportion

Two values are said to be in direct proportion when the following conditions are met:

- Each time when the value of the former is increased, the latter will increase as well.
- Both values can be expressed in the following form $y = bx$, where b is a constant.

(Some teachers and questions may use different letters but they essentially mean the same thing.)

Example 1:
y is directly proportional to x. Given that $y = 144$ and $x = 12$. Find the value of y when $x = 7$.

$y_{given} = 144$
$x_{given} = 12$
Since y is directly proportional to x,
$y_{given} = b(x_{given})$
$144 = b(12)$
$b = 12$

Since $x_{new} = 7$
$y_{new} = b(x_{new})$
$y_{new} = 12(7)$
$y_{new} = 84$

When $x = 7, y = 84$

Inverse Proportion

Two values are said to be in inverse proportion when the following conditions are met:

- Each time when you increase the former value, the latter value decreases. (And each time when decrease the former value, the latter value increases.)
- Both values can be expressed in the form of $y = \frac{b}{x}$

Example 2:

L is inversely proportional to the $\sqrt{M}$. When $M = 100, L = 35$.
Find the value of M, when $L = 700$.

$L_{given} = 35$
$M_{given} = 100$
$\sqrt{M}_{given} = \sqrt{100} = 10$

$$L_{given} = \frac{b}{\sqrt{M}_{given}}$$

$$35 = \frac{b}{\sqrt{100}}$$

$$b = 35(10) = 350$$

Since $L_{new} = 700$

$$700 = \frac{350}{\sqrt{M}_{new}}$$

$$700\sqrt{M}_{new} = 350$$

$$\sqrt{M}_{new} = \frac{350}{700} = \frac{1}{2}$$

$$M_{new} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

Title	Mathematics (Maps and Scales)
Date	20/3/2018
Author	Lim Wang Sheng, School of Information Technology, Nanyang Polytechnic Lee Jian Lian (Editor)

Logic and terms that you need to understand before proceeding:

Why study this topic?

This topic, not surprisingly, has many practical applications. That's precisely how people in the past all the way till now, create maps and allow users of the map to understand, how far should you go from one place to another when you are only just given a map.

Even in the modern days, when smartphones map applications are gradually taking over, that's exactly how smartphones can tell you how long you are supposed to walk on a street to get to another place. The map application is set to interpret a certain "zoom" level as a certain scale on a map, zooming in the map reduces ratio between literal map distance and actual distance, zooming out the map does the opposite.

Terms used in this topic	**Meaning**
D_{map}	The literal distance of any two mentioned points within a map.
D_{actual}	The actual distance of any two mentioned points within a map, in real life.
A_{map}	The literal area of any shape as displayed on a map.
A_{actual}	The actual area of any mentioned shape within a map, in real life.

Formula List

Scale of Map Formula (In representative fraction)	Scale = $\frac{D_{map}}{D_{actual}}$ (Where D_{map} *has to be reduced to* 1 *to be expressed as a fraction*.
Ratio of areas between map area and actual area.	$\left(\frac{A_{map}}{A_{actual}}\right) = \left(\frac{D_{map}}{D_{actual}}\right)^2$

Questions All Taken from CASCO Mathematics 4B Assessment

Example 1:

On a map, a length of 4 cm represents an actual distance of 1 km.

Calculate

a) the actual distance, in kilometers, represented by 26 cm on the map,
b) the scale of the map in the form of $1{:}\,n$,
c) the area on the map, in square centimeters, which represent an actual area of $9\ km^2$

To solve this question, we need to derive the following information

Details	**Steps Taken**
Ratio between map distance and actual distance in real life	Scale = $\frac{D_{map}}{D_{actual}}$ = $\frac{4cm}{1km} = \frac{1\ cm}{0.25\ km}$ We must make sure the units are the same to establish the scale's representative fraction: $\frac{1\ cm}{250\ m} = \frac{1\ cm}{25000\ cm}$ Remove the units to get Scale = $\frac{1}{25000}$
Ratio between map area and actual area in real life	Area Scale (Without Unit) = $\left(\frac{A_{map}}{A_{actual}}\right) = \left(\frac{D_{map}}{D_{actual}}\right)^2 = \left(\frac{1}{25000}\right)^2$ Area Scale (With Unit) = $\left(\frac{1\ cm}{0.25\ km}\right)^2 = \frac{1\ cm^2}{0.0625\ km^2}$

a)

Let y be the actual distance in real life.

Given Scale Fraction = $\frac{1\ cm}{0.25\ km}$ = $\frac{26\ cm}{y\ km}$

$y = 26 \times 0.25 = 6.5$

Thus, distance in real life = 6.5 km

b)

Given Scale Fraction = $\frac{1}{25000}$

When written in ratio form it is $1{:}\,25000$

c)

Given Area Scale Fraction (With Unit) = $\frac{1\ cm^2}{0.0625\ km^2}$

Let x be the area on map

$$\frac{x\ cm^2}{9\ km^2} = \frac{1\ cm^2}{0.0625\ km^2}$$

$$x = \frac{9}{0.0625} = 144$$

Thus, area on map = 144 cm^2

Title	Mathematics (Solving Quadratic Equations)
Author	Lim Wang Sheng, School of Information Technology, Nanyang Polytechnic [CCA: NYP Mentoring Club]
Date	20/3/2018

Mathematics Terms
"Root", "Solution" and "Answers" means the same thing in this topic. They are a set of numbers that can satisfy a specified equation.
Quadratic Equation – an equation for which the highest power within the equation is 2.
LHS – Left-hand Side
RHS – Right-hand Side
"Find Solution" and "Solving" mean the same thing in this topic.

Identifying a quadratic equation

Based on the above definition of a quadratic equation, a quadratic equation is any equation that is, in the following form or can be re-arranged in the following form:

$$ax^2 + bx + c = 0$$

Where $a \neq 0$

[If $a = 0$, the equation effectively reduces to $bx + c = 0$, which is linear, not quadratic.]

Three approaches used to solve quadratic equation I am going to teach

- Factorization/Cross Method (Calculator Tips and Tricks Provided as well)
- Quadratic Formula
- Completing the Square Method

Factorization/Cross Method

Algebraic Identity Approach

Only work when the given equation resembles any 1 of the 3 algebraic identity provided as below.

- $a^2 + 2ab + b^2 = 0$
- $a^2 - 2ab + b^2 = 0$
- $a^2 - b^2 = 0$

As all the expression on the LHS (Left-Hand side) of the equation can be factored into the following:

- $a^2 + 2ab + b^2 = (a + b)^2$
- $a^2 - 2ab + b^2 = (a - b)^2$
- $a^2 - b^2 = (a + b)(a - b)$

Basic Demonstration

Example 1

Solve $x^2 + 6x + 9 = 0$

Noting how the above equation resembles the expression:

$$a^2 + 2ab + b^2 = (a + b)^2$$

As $2ab = 2(x)(3) = 6x$

Factorize the LHS of the equation into

$x^2 + 6x + 9 = (x + 3)^2$

$(x + 3)^2 = 0$

$(x + 3)(x + 3) = 0$

$x + 3 = 0\ OR\ x + 3 = 0$

Final Answer: $x = -3$

Example 2:
Solve $x^2 - 8x + 12 = -4$

In this case, the RHS of the equation isn't zero, so, we have to rearrange the equation in the following way:
$x^2 - 8x + 12 + 4 = -4 + 4$
$x^2 - 8x + 16 = 0$

We can tell the above expression on the LHS of the equation resembles the following:

$$a^2 - 2ab + b^2 = (a-b)^2$$

As $(-2ab) = -2(x)(4) = -8x$

Factor the LHS of the equation into the following:
$x^2 - 8x + 16 = (x-4)^2$
$(x-4)(x-4) = 0$
$x = 4\ OR\ x = 4$

Final Answer: $x = 4$

Example 3:
Solve $x^2 - 64 = 0$

We can tell the above expression on the LHS of the equation resembles the following:

$$a^2 - b^2 = (a+b)(a-b)$$

$a^2 = x^2$
$-b^2 = -64$

Thus, the expression can be factored in the following:
$x^2 - 64 = (x+8)(x-8)$

$(x+8)(x-8) = 0$
$x + 8 = 0\ OR\ x - 8 = 0$
Final Answer: $x = -8\ OR\ x = 8$

Cross Method (Calculator Tricks)
Look out for any options on your calculator that can help you to solve any quadratic equations by keying in values $a, b\ and\ c$ as you will need to use it.

In the case of FX-95SG-PLUS
You press “Mode”, “3” “3” to have access to such functionality, other calculators may have similar functionality.

Troubleshooting and Error Messages (Using FX-95 SG-PLUS calculator as example)
If you get a “Math Error” message, check the following:
Did you put $a = 0$? [a cannot be 0]
Did you key in value of a wrongly as something else which is not number?

If you get “Syntax Error” message, check the following:
Make sure you didn’t accidentally type in any symbols apart from numbers into the calculator in such mode.

Example 4:
Solve $x^2 - 7x - 330 = 0$
[Due to technical difficulties, it is very difficult to draw and demonstrate cross method on computer. I investigated in methods to do so but none of them produces a nice, decently looking drawing. I want to remind students here, if you are using calculator to solve quadratic equation directly, you must also demonstrate how it can be done using cross method, quadratic formula or completing the square on your question paper or answering booklet. Failure to include the method of doing results in loss of marks.]

After keying in the value of $a = 1, b = -7\ and\ c = -330$ into the calculator
I get the following values as shown:

Final Answer
$x = -15\ OR\ x = 22$

Quadratic Formula Approach

As the name suggest, quadratic formula is a formula based method used to solve quadratic equation, it is (sort of) derived from completing the squares which I will share with you in the subsequent pages.

In any quadratic equation that you are trying to solve, there can be 2 real and distinct roots, equal roots or no real roots, this would depend on the discriminant $b^2 - 4ac$ (which I will not further discuss here as discriminant related problems appear more in Additional Mathematics question papers, as compared to Elementary Mathematics.)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 5

Solve $6x^2 + 7x - 15 = 0$

In this case, you are going to substitute the quadratic formula with coefficient values shown in the equation.

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad OR\ x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-7 + \sqrt{7^2 - 4(6)(-15)}}{2(6)} \quad OR\ x = \frac{-7 - \sqrt{7^2 - 4(6)(-15)}}{2(6)}$$

$$x = \frac{-7}{12} + \frac{\sqrt{409}}{12} \quad OR \quad x = -\frac{7}{12} - \frac{\sqrt{409}}{12}$$

$$x = 1.10\ OR\ x = -2.27\ (3sf)$$

Example 6

Solve $x^2 + x + 1 = 0$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(1)}}{2(1)}$$

$$x = \frac{-1 + \sqrt{1^2 - 4(1)(1)}}{2(1)} \quad OR\ x = \frac{-1 - \sqrt{1^2 - 4(1)(1)}}{2(1)}$$

Situation (On FX-95 SG Plus)	What to expect when such an attempt to solve the equation is made.
You solve equation manually (i.e. key in one by one).	"Math Error" Message.
You solve it using the quadratic equation solver functionality within the calculator	Numbers that comes in the form of $a + bi$ In this case, I get the following values: $x = -0.5 + 0.866i\ OR\ x = -0.5 - 0.866i$
**Different calculator operates differently.	

Respond to both answers by writing the following on your answering booklet

Final Answer: No Real Roots.

Completing the squares approach

Example 7
Solve $x^2 + 7x + 9 = 0$

Rearrange equation into the following format:
$(x \pm h)^2 \pm k = 0$

Which means you have to do the following steps:
Add $\left(\frac{c}{2}\right)^2$ between bx and c and Subtract $\left(\frac{c}{2}\right)^2$ after c,
which can be shown as follows:

$$x^2 + 7x + \left(\frac{7}{2}\right)^2 + 9 - \left(\frac{7}{2}\right)^2 = 0$$

$$\left(x + \frac{7}{2}\right)^2 - 3.25 = 0$$

$$\left(x + \frac{7}{2}\right)^2 = 3.25$$

$$\left(x + \frac{7}{2}\right) = \pm\sqrt{3.25}$$

$$x + 3.5 = \sqrt{3.25} \quad OR\ x + 3.5 = -\sqrt{3.25}$$

$$x = -\sqrt{3.25} - 3.5\ OR\ x = \sqrt{3.25} - 3.5$$

$$x = -5.30\ OR\ x = -1.70\ (3sf)$$

Example 8

Special case of Completing the Square where $a > 1$ or $a < -1$

Solve $7x^2 - 22x + 10 = 0$

We need to first, divide LHS of the equation by a.

$$\frac{7x^2 - 22x + 10}{7} = 0$$

$$x^2 - \frac{22}{7}x + \frac{10}{7} = 0$$

In this case, we have to add $\left(\frac{c}{2a}\right)^2$ between $\frac{b}{a}$ and $\frac{c}{a}$ and subtract $\left(\frac{c}{2a}\right)^2$ after $\frac{c}{a}$.

$$x^2 - \frac{22}{7} + \left(\frac{\frac{22}{7}}{2}\right)^2 + \frac{10}{7} - \left(\frac{\frac{22}{7}}{2}\right)^2 = 0$$

$$x^2 - \frac{22}{7} + \left(\frac{22}{14}\right)^2 + \frac{10}{7} - \left(\frac{22}{14}\right)^2 = 0$$

$$\left(x - \frac{22}{14}\right)^2 - \frac{51}{49} = 0$$

$$x - \frac{22}{14} = \pm\sqrt{\frac{51}{49}}$$

$$x = \sqrt{\frac{51}{49}} + \frac{22}{14}\ OR\ x = -\sqrt{\frac{51}{49}} + \frac{22}{14}$$

Final Answer:

$x = 2.59\ or\ x = 0.551\ (3sf)$

Title	Mathematics (Solving Simultaneous Equations) [Secondary 2]
Editor	Lee Jian Lian Originally By: Lim Wang Sheng, School of Information Technology, Nanyang Polytechnic [NYP Mentoring Club]
Date	22/3/2018

[Just in case you feel surprised by this release, this version is an edited version of past version I created, with the Additional Mathematics component of the guide to solving simultaneous equation removed.]

When a question is asking you to solve a simultaneous equation, these are the most common approaches I have seen for levels up to Secondary 5:

- Elimination Approach
- Substitution Approach

I am going to demonstrate how to use these two methods to solve simultaneous equations and will even explain on why either of the methods is suitable for various situations.

Elimination Approach
Question 1
Solve the simultaneous equations:

$2x - 5y = -1$	Equation 1
$6x - 4y = 30$	Equation 2

Why I think elimination method is more suitable for this question.

- Done in fewer steps (Just multiply by their common factors)
- The question creates a situation that is more favorable to using elimination method.

From Equation 1 to Equation 1A
$3(2x - 5y) = 3(-1)$
$6x - 15y = -3$

Equation 2 - Equation 1A
$(6x - 4y) - (6x - 15y) = 30 - (-3)$
$6x - 4y - 6x + 15y = 33$
$-4y + 15y = 33$
$11y = 33$

Equation 3
$y = 3$

Substitute Equation 3 into Equation 1
$2x - 5(3) = -1$
$2x = -1 + 15$
$2x = 14$
$x = 7$

Answer: $y = 3, x = 7$

Substitution Approach
Question 2
Solve the following simultaneous equations.

$y = 2x - 5$	Equation 1
$7x - 4y - 12 = 0$	Equation 2

Why I think substitution method is more suitable in this case

- It takes less effort to substitute the values as you just need to substitute equation 1 into equation 2.

Substitute Equation 1 into Equation 2.
$7x - 4(2x - 5) - 12 = 0$
$7x - 8x + 20 - 12 = 0$
$-x + 8 = 0$
$-x = -8$

Equation 3
$x = 8$

Substitute Equation 3 into Equation 1.
$y = 2(8) - 5$
$y = 16 - 5 = 11$

$y = 11$

$x = 8, y = 11$

Title	**Solving Problem Sums with Quadratic Equations**
Author	Lim Wang Sheng, School of Information Technology, Nanyang Polytechnic [From Nanyang Polytechnic Mentoring Club]
Editor	Hui Ling, Ngee Ann Polytechnic
Inspector	Lee Jian Lian
Date	26/2/2018

I am Wang Sheng from Mentoring Club. I am here to demonstrate how do you solve a problem sum by using quadratic equation. The procedure if you notice, it is standardized among all types of questions requiring you to form quadratic equation and solve them to get your answer.

[DO NOT READ THIS DOCUMENT IF YOU DON'T UNDERTAND HOW TO FIND SOLUTIONS TO A QUADRATIC EQUATION. THIS DOCUMENT ASSUMES YOU UNDERSTAND COMPLETING THE SQUARES AND QUADRATIC FORMULA.]

Step	Description
1	Form Expressions
2	Form Equation
3	Reduce the Equation to Quadratic Form
4	Find the Solutions to the Quadratic Equation(s) derived
5	Reject Values that doesn't make sense. (Examples: Dividing by Zero, Negative Distance.) [If Applicable]
6	Answer Remaining Questions

The six steps will be demonstrated in the next page and these are based on my teaching experience in my CCA. I will try to make the whole document as simple to understand as possible for beginners.

[Note: Questions Taken from CASCO Mathematics Assessment Book 4B]

Example Question 1:

A motorist travelled 60km from P to Q at an average speed of x km/h.

(a) Write down an expression, in terms of x, for the time taken, in hours, for the journey.

On his return journey from Q to P, his average speed was reduced by 5 km/h due to heavy traffic on the way.

(b) Write down an expression, in terms of x, for the time taken, in hours, for the return journey.

(c) If the return journey took 10 minutes longer, form an equation in x and show that it reduces to

$$x^2 - 5x - 1800 = 0$$

(d) Solve the equation $x^2 - 5x - 1800 = 0$

(e) Find the time taken for the return journey.

Step 1. Form Expression

Q1(a)	Distance = 60km Average Speed = x km/h Since $\frac{distance}{speed} = time$ Time taken from P to Q must be = $\frac{60}{x}\ hours$.
Q1(b)	Distance = 60 km (Reverse Direction Thus Distance Unchanged) Speed = $x - 5$ km/h. Since $\frac{distance}{speed} = time$ Time taken from Q to P must be = $\frac{60}{x-5}$ hours.

Step 2. Form Equation

Q1(c)	Since the return journey is 10 minutes longer, and the unit specified in the question is in hours, we must convert “10 minutes” into $\frac{1}{6}$ hours. Therefore, the following equation is formed. (Take note, the return journey is longer, so the value must be, return journey – the initial trip = $\frac{1}{6}$ hours. $$\frac{60}{x-5} - \frac{60}{x} = \frac{1}{6}$$

Step 3. Show that the equation can be reduced into quadratic form as shown in the question.

Q1(c)	$\frac{60}{x-5} - \frac{60}{x} = \frac{1}{6}$ $\frac{60(x)}{x(x-5)} - \frac{60(x-5)}{x(x-5)} = \frac{1}{6}$ $\frac{60x - 60x + 300}{x^2 - 5x} = \frac{1}{6}$ $\frac{(60x - 60x + 300)}{x^2 - 5x} = \frac{1}{6}$ $\frac{300}{x^2 - 5x} = \frac{1}{6}$ $300 = \frac{1}{6}(x^2 - 5x)$ $6(300) = x^2 - 5x$ $0 = x^2 - 5x - 1800$ $x^2 - 5x - 1800 = 0$ [Shown]
Notes	TAKE NOTE OF NEGATIVE SIGNS

Step 4. Find Solution to the Equation (Also called solve the equation)

Q1(d)	(In this case, equation will be solved using quadratic formula method, if the question doesn't specify any methods, you can use any method you like.) However, if the question does require specific methods to be used to solve the equation, use the stated method in the question. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $a = 1, b = -5, c = -1800$

	$x = \dfrac{-(-5) + \sqrt{(-5)^2 - 4(1)(-1800)}}{2(1)}$ OR $x = \dfrac{-(-5) - \sqrt{(-5)^2 - 4(1)(-1800)}}{2(1)}$ $x = 45\ OR\ x = -40$

Q1(e)	Since you can't have negative speed value, $x = -40$ must be rejected. Since question asks for time required for return journey. They are asking for the value of $\dfrac{60}{x-5}$ Substitute $x = 45$ into the above expression to get $\dfrac{60}{(45-5)} = 1.5$ Time taken for return journey = $1\ hour\ and\ 30\ minutes.$

Title	Mathematics (Simplifying Indices Expressions)
Author	Lim Wang Sheng, School of Information Technology, Nanyang Polytechnic [From NYP Mentoring Club]
Editor	Lee Jian Lian
Date	2/3/2018

Students doing indices questions may find difficulties in the following areas and this document aim to address all the areas:

- Basic indices concepts (In detail most MOE teachers will not get into unless in a personal remedial lesson)
- Complicated indices operations (Fraction inside fraction, powers and negative power in one expression, along with multiple brackets)

Let's look at the various laws of indices we must remember before proceeding.

Basic Laws of Indices	Simplest Proof of Concept
$a^m(a^n) = a^{m+n}$	$2^2 \times 2^3 = (2 \times 2) \times (2 \times 2 \times 2) = 2^5$
$\frac{a^m}{a^n} = a^{m-n}$	$\frac{2^2}{2^1} = (2 \times 2) \div 2 = 2^1$
$(a^m)^n = a^{mn}$	$(2^3)^2 = (2 \times 2 \times 2) \times (2 \times 2 \times 2) = 2^6$
$a^{-n} = \frac{1}{a^n}$	$\frac{2^2}{2^4} = \frac{2\times2}{2\times2\times2\times2} = \frac{1}{2^2} = 2^{-2}$
a^0 =1 (Where $a \neq 0$)	Reasoning: $a^1 = a$ Thus $\frac{a}{a} = a^{1-1} = a^0 = 1$
$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$\left(\frac{2}{5}\right)^3 = \left(\frac{2}{5}\right)\left(\frac{2}{5}\right)\left(\frac{2}{5}\right) = \frac{2 \times 2 \times 2}{5 \times 5 \times 5} = \frac{2^3}{5^3}$
$a^{\frac{m}{n}} = \sqrt[n]{a^m}$	Arithmetic Reasoning: $(2^4) = 2 \times 2 \times 2 \times 2$ $\sqrt{2 \times 2 \times 2 \times 2} = 2 \times 2 = 2^{\frac{4}{2}} = 2^2$
$(ab)^n = a^n b^n$	

Derived Law of Indices	Simplest Proof of Concept
$\left(\frac{a^m}{a^n}\right)^{-1} = \frac{a^n}{a^m}$	$\left(\frac{1}{5}\right)^{-1} = \frac{1}{\left(\frac{1}{5}\right)} = \frac{5}{1} = 5$
$\left(\frac{a^m}{a^n}\right)^{-x} = \left(\frac{a^n}{a^m}\right)^{x}$	

Tips to do complicated indices question (with nested fractions, powers and negative powers all in the same question.)

Read question (Examine the details closely)
Take note of any "out of the ordinary" situation (Example can include zero powers, that makes solving a seamlessly complicated question easier)
Take note of any negative powers.
Take note of any roots.
Write the expression in fully index notation
Solve the question from inner brackets to outer bracket
Check your answers

I will demonstrate some questions and explain how the tips can be applied.

(Questions all taken from CASCO Mathematics Assessment Book 4B)

Example 1:

Simplify $(4x^2)^{\frac{3}{2}}$

$(4x^2)^{\frac{3}{2}}$	(Things to take note highlighted)
$= 4^{\frac{3}{2}}x^{2\left(\frac{3}{2}\right)}$	Apply the following laws of indices $a^{\frac{m}{n}} = \sqrt[n]{a^m}$ $(ab)^n = a^n b^n$
$= 8x^3$	(Final Answers)

Example 2:

Evaluate $\left(\frac{1}{2}\right)^{-4} \times 8^0$

$\left(\frac{1}{2}\right)^{-4} \times 8^0$	Things to take note (Zero Power and Negative Powers)
$= \left(\frac{2}{1}\right)^{4} \times 1$	Apply the following law of indices $a^0 = 1$ $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
$= 16$	

Example 3:

Given that $\frac{(x^2)^3 \times \sqrt{x}}{\sqrt[3]{x}} = x^n$. Calculate the value of n.

$\frac{(x^2)^3 \times x^{\frac{1}{2}}}{x^{\frac{1}{3}}} = x^n$	Convert values all to index notation
$\frac{(x^6 \times x^{\frac{1}{2}})}{x^{\frac{1}{3}}} = x^n$ $\frac{x^{6+\frac{1}{2}}}{x^{\frac{1}{3}}} = x^n$	Apply the following law of indices $(a^m)^n = a^{mn}$ $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
$x^{6\frac{1}{2}-\frac{1}{3}} = x^{6\frac{1}{6}} = x^n$	Apply the following law of indices $\frac{a^m}{a^n} = a^{m-n}$

$n = 6\frac{1}{6}$	